TRANSIENT PROCESSES OF FLOW OF ELASTOVISCOPLASTIC MEDIA IN LONG CHANNELS

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The article deals with the special features of the rheodynamics and thermophysics of the shear strain of elastoviscoplastic media in long channels when a pressure gradient is suddenly imposed.

The widely used liquid disperse systems of the type of filled polymer solutions, clayey suspensions, drilling fluids are able to form spatial structures from particles of the disperse phase. The rheology of such systems is distinguished by a number of special features. At stresses below yield point the structure is an elastically deformed skeleton in solid form. At stresses exceeding the yield point, flow with reversible rupture of contacts between elements of the structure sets in. The existence of a yield point, of elastic properties, and also of effects of viscosity, creep, stress relaxation manifesting themselves in flow determines the peculiarities of the dynamic behavior of the media under consideration. On account of the spatial inhomogeneity of the stress fields in them, zones with ruptured and nonruptured structure form in them. A change of the loading conditions causes relative displacement of the interfaces between regions with different states of the structure, and in the zone with nonruptured structure it gives rise to elastic waves.

Let us consider a simple model situation. A medium at rest in a long flat channel is suddenly subjected to a longitudinal pressure gradient $\partial p/\partial z = (\partial p/\partial z)_0 \cdot 1(t)$. The response reaction to this effect depends on the magnitude of the yield point, the width of the channel, and the pressure gradient. If the mechanical action does not destroy the spatial structure, then only reversible deformations are induced in it, and its mechanical behavior is described by the equations of motion of an elastic finitely deformed medium. For a flat channel and simple shear

$$\rho \frac{\partial^2 U}{\partial t^2} = -\frac{\partial p}{\partial z} + G \frac{\partial^2 U}{\partial y^2}. \tag{1}$$

The solution of Eq. (1) with zero initial conditions and with adhesion of the medium to the wall determines the dependence of the tangential stress at the channel wall on time [1]

$$\tau_w = \left[H\left(\frac{\partial p}{\partial z}\right)_0 / T_{\text{els}}\right][t\cdot 1(t) - 2(t - T_{\text{els}})\cdot 1(t - T_{\text{els}}) + \ldots]. \tag{2}$$

The maximal stress at the wall is $\tau_{\max}^{\text{dyn}}$. When the maximal stresses are lower than the yield point, oscillations of the structure without dissipation are effected. If the external loads induce stresses greater than the yield point ($H(\partial p/\partial z)_0 > Y$), then the deformation is accompanied by dissipation of mechanical energy in the near-wall zones of the channel where flow of the destroyed structure occurs. This is a transient process of deformation from rest to a new steady state. The result of the transient process is determined solely by the stress level, and the features of its course by the stress level and by the intensity of dissipation.

After impulsive change of the pressure gradient in the channel, the stresses begin to increase. In the near-wall region they overlap the yield point and form a zone of flow whose evolution in dependence on the external conditions may proceed in two directions. One possible variant is the gradual attainment of a regime of steady flow in the region with destroyed structure in the time $T_{\text{vis}} \sim \rho H^2/\eta$ which, with fixed level of yield stress, is inversely proportional to the rate of dissipation of mechanical energy. It follows from the law

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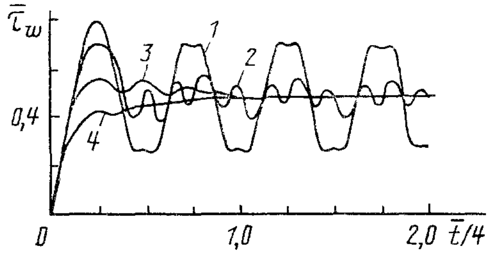


Fig. 1

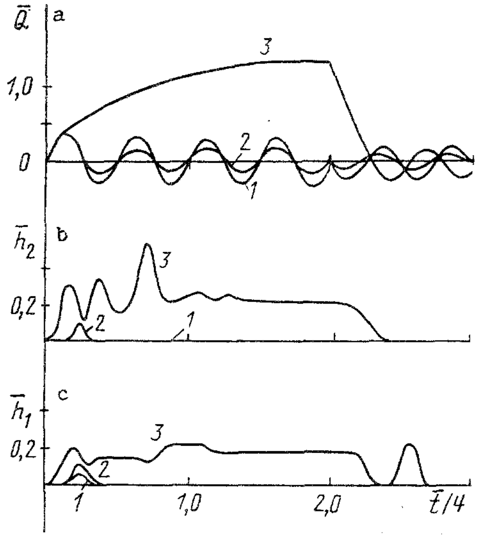


Fig. 2

Fig. 1. Time-dependent change of stresses on the wall of a flat channel (δ = 99, ζ = 50): 1) β_0 = 0.75; 2) 0.6; 3) 0.45; 4) 0.3.

Fig. 2. Time-dependent change of the volumetric flow rate in a coaxially cylindrical channel (a) and of the width of the flow zone with destroyed structure at the outer wall (b) and at the inner wall (c) upon increase and drop ($\bar{t}$ = 8) of the pressure gradient: 1) β_0 = 0.9; 2) 0.7; 3) 0.3. The curvature of the channel δ = 1, ζ = 50.

of conservation of momentum that with steady flow at the wall, the tangential stress is maximal and equal to $\tau^{st}_{max} = H(\partial p/\partial z)_0/2$. That means that transition to the regime of steady flow of the destroyed structure is possible only on condition that $H(\partial p/\partial z)_0/2 \geq Y$. With $\tau^{st}_{max} < Y < \tau^{(dyn)}_{max}$ the transient process proceeds with the inclusion and exclusion of the dissipative mechanism at instants when the yield point is exceeded (at the approach of the elastic wave) and when the stress is below Y (at the departure of the wave), respectively. Energy dissipation in a region with destroyed structure gradually reduces the amplitude of the stress oscillations in the wave. This type of "dynamic" transient process is completed when the maximal tangential stress in the wave is below the yield point. The structure of the medium is then not subject to further destruction. An oscillation regime is then established of cumulation and release of the elastic energy of reversible deformations of the undestroyed structure. We note that the duration of the "dynamic" transient regime is determined by the rate of dissipation in the appearing zones with destroyed structure. The higher the rate, the more rapidly decreases the amplitude of the stress oscillations in the wave to a value below the yield point. Thus there are possible two types of transient regime of shear strain in the systems under consideration. In the present work we study them for channels with flat and annular profile. We also examine the general features of the course of thermal processes in similar media.

For a qualitative investigation of systems with the above-described complex of structural and mechanical properties we chose the model of an elastoviscoplastic medium which for nonsteady shear flow has the form

$$\left[\frac{\tau - Y\,\mathrm{sign}\,(\dot{\gamma})}{\eta}\right]\cdot 1\,(|\tau| - Y) + \frac{1}{G}\,\frac{\partial \tau}{\partial t}\cdot 1\,(Y - |\tau|) = \dot{\gamma}. \quad (3)$$

This model does not take into account the effects of creep and stress relaxation but it makes it possible to reveal and evaluate the main peculiarities of transient processes in elastoviscoplastic systems, also in thixotropic media where the times of destruction and restoration of the structure may be neglected [2, 3].

The mechanical properties of a medium with the rheological equation of state (3) are conveniently characterized according to its behavior in regimes of uniform deformation. With the instantaneous change of the strain rate $\dot{\gamma} = \dot{\gamma}_0 \cdot 1(t)$, $\dot{\gamma}_0 > 0$, the stresses increase linearly with time as long as $\tau < Y$. When $\tau = Y$, the stresses increase jumplike by the magnitude of the viscous component $\eta\dot{\gamma}_0$ and remain constant until the strain rate has vanished ($\dot{\gamma}$= 0). At that instant the yield point Y is attained. At an instantaneous change of stresses

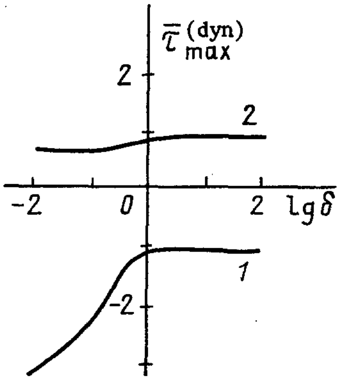


Fig. 3

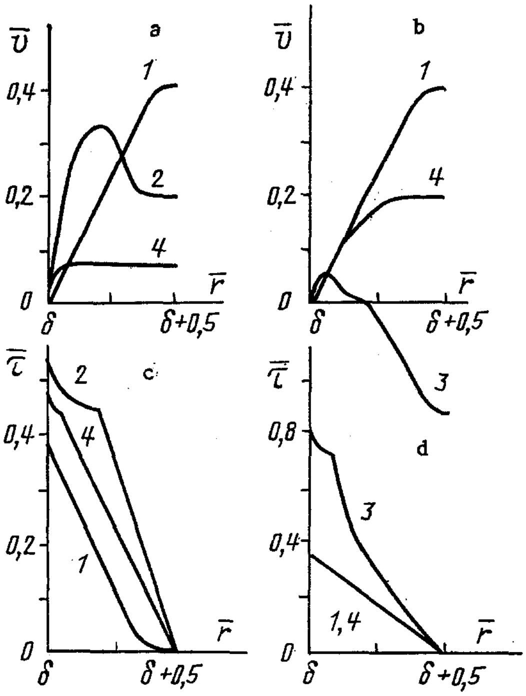


Fig. 4

Fig. 3. Dependence of the maximal dynamic stress at the inner wall (curve 1) and at the outer wall (curve 2) on the curvature of the channel.

Fig. 4. Change of the speed (a, b) and of the stress (c, d) over the section of a flat channel, $\zeta = 50$: 1) $\bar{t}/4 = 0.1$; 2) 0.2; 3) 0.3; 4) 1.6; $\beta_0 = 0.45$ (a, c), $\beta_0 = 0.75$ (b, d).

$\tau = \tau_0 \cdot 1(t)$, $\tau_0 > Y$, there follows an instantaneous response of inverse strain $\gamma_0 = Y/G$ and increase of deformation that is linear with time. When the stress drops to zero, deformation decreases to the value γ_0.

For hydrodynamic transient processes relation (3) is supplemented by the equation of motion

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau), \tag{4}$$

and also by the initial and boundary conditions:

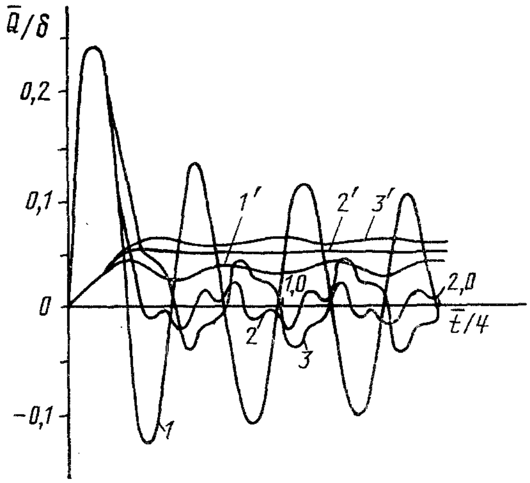
$$v|_{t=0} = 0, \tag{5}$$

$$v|_{r=R_1} = 0, \quad v|_{r=R_2} = 0. \tag{6}$$

The initial-boundary value problem (3)-(6) may be written in dimensionless variables. It contains two independent parameters $\bar{r} = r/H$, $\bar{t} = t/T_{els}$ and three dimensionless complexes $\beta_0 = Y/(H(\partial p/\partial z)_0)$, $\beta_1 = G/(H(\partial p/\partial z)_0)$, $\zeta = H\sqrt{\rho G}/\eta$. The criterion β_0 determines the type of transient regime. Specifically, for a flat channel with $0.5 < \beta_0 < 1$ oscillations of stresses and strains are realized (the "dynamic" type of transient regime). When $0 < \beta_0 < 0.5$, then as a result of the transient process two regions form: at the walls, where we find steady viscoplastic flow with completely destroyed structure, and at the axis where the medium with intact structure moves as a whole. The complex ζ determines the ratio of the characteristic times of propagation of the viscous and of the elastic shear wave: $\zeta = T_{vis}/T_{els}$. The parameter β_1 is the measure of elastic deformations in the region with intact structure $\beta_1 \sim \gamma^{-1}$. The value of β_1 is henceforth fixed and equal to 1.

For calculating heat exchange we use the equation of convective energy transfer

$$\rho \frac{Du}{Dt} = -\nabla\bar{q} + \underset{\sim}{T} : \underset{\sim}{D}, \ \bar{q} = -\lambda\nabla\theta, \tag{7}$$



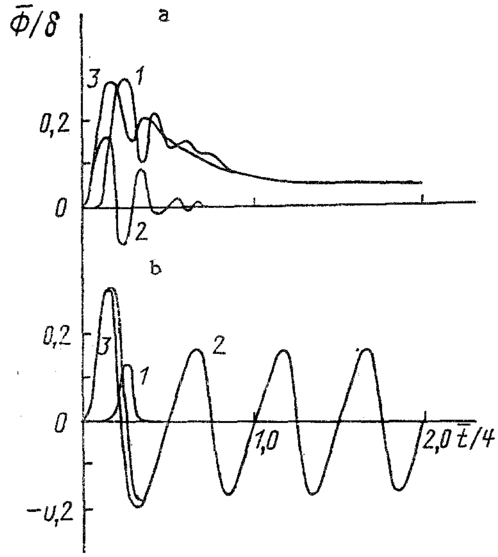


Fig. 5 Fig. 6

Fig. 5. Change of the instantaneous volumetric flow rate (curves 1-3) and of the total volume of the medium passing through the cross section of a flat channel (curves 1'-3') in dependence on the parameter ζ: 1, 1') $\zeta = 5$; 2, 2') 50; 3, 3') 500. $\beta_0 = 0.6$, $\delta = 99$.

Fig. 6. Time-dependent change of the dissipative function $\bar{\Phi} = \Phi/\nu$, averaged over the zone with destroyed structure $\bar{\Phi}_2$ (curve 1), with intact structure $\bar{\Phi}_1$ (curve 2), over the entire cross section of the flat channel $\bar{\Phi}$ (curve 3); a) $\beta_0 = 0.45$; b) 0.75. $\zeta = 50$, $\delta = 99$.

jointly with the energy equation of state. In the region with destroyed structure, where the deformations are irreversible, the internal energy is a function of the temperature only, $u = u(\theta)$, and Eq. (7) turns into an ordinary equation of heat transfer for inelastic liquids

$$\rho c_p \frac{D\theta}{Dt} = \nabla \bar{q} + \underset{\sim}{T} : \underset{\sim}{D}. \tag{8}$$

Equation (8) makes it possible in principle to calculate a thermal problem when the velocity field and the space-and-time distribution of the dissipative heat releases in the channel are known. Below these data are presented from the numerical solutions of the system (4)-(6).

Let us deal with the region with intact structure. The equation of free energy for systems with elastic properties in accordance with the principle of equal presence contains as independent variables the temperature and the strain tensor: $A = A(\theta, \underset{\sim}{F}_R)$ [4]. In elastic systems the deformations are usually correlated with a change of internal energy $u = u(\theta, \underset{\sim}{F}_R)$. We isolate the equilibrium part of the internal energy of the systems whose change occurs solely on account of a change of temperature. Then $u = u_0(\theta) + \Delta u(\theta, \underset{\sim}{F}_R)$ and it follows from (7) that

$$\rho (c_p + \Delta c_p) \frac{D\theta}{Dt} = -\nabla \bar{q} + \left(\underset{\sim}{T} - \rho \underset{\sim}{F}_R \left(\frac{\partial u}{\partial \underset{\sim}{F}_R}\right)^{\mathrm{T}}\right) : \underset{\sim}{D}, \tag{9}$$

Δc_p denotes the contribution of elastic deformations to the heat capacity of the medium; usually it is small compared with c_p.

Let us consider two limit cases. In polymers media deformations are connected chiefly with a change of the configurational entropy of the polymer chains only: $s = s(\theta, \underset{\sim}{F}_R)$. They do not affect the magnitude of the internal strain energy: $u = u(\theta)$. If we assume that in the region with intact structure there is in fact only entropy elasticity, then we obtain from Eq. (9) the equation of temperature distribution in the form (8).

The second law of thermodynamics correlates the equation of free energy with the rheological equation of state of a system that is suitable for reversible deformations [5]:

$$\underset{\sim}{T} = \rho \underset{\sim}{F}_R \left(\frac{\partial A}{\partial \underset{\sim}{F}_R}\right)^{\mathrm{T}}. \tag{10}$$

Substituting (10) into (9), we obtain

$$\rho c_p \frac{D\theta}{Dt} = -\nabla\bar{q} + \rho\theta F_R \left(\frac{\partial s}{\partial F_R}\right)^{\tau} : D. \tag{11}$$

In elastic solids the deformations are of a purely energetic nature: $u = u(\theta, F_R)$. Entropy does not depend on deformations: $s = s(\theta)$ [5]. If we regard these relations as correct for the region with intact structure, then a change of temperature is determined by the ordinary equation of heat conduction

$$\rho c_p \frac{D\theta}{Dt} = -\nabla\bar{q}. \tag{12}$$

Thus, in the examined limit cases of purely entropy and purely energetic elasticity there are equations for the temperature which make it possible to model the processes of heat transfer in the zone with intact structure. If in the state with intact structure the medium has entropy elasticity, then in the entire region of deformation the equation of heat transfer has the form (8), and for the condition of energetic elasticity in the region with destroyed structure an equation type (8) has to be used, in the region with intact structure Eq. (12) has to be used.

For the flow of liquid disperse systems in long mains with slow heat liberation we may use the equation of heat balance [6]

$$\rho c_p \frac{d\bar{\theta}}{dt} = q_2 + q_1 + \Phi,$$
$$\bar{\theta} = 2\pi \int_{R_1}^{R_2} r\theta dr, \quad \Phi = 2\pi \int_{R_1}^{R_2} \tau r (\partial v/\partial r)\, dr. \tag{13}$$

Here, Φ is a dissipative function averaged over the entire range of deformation when $u = u(\theta)$, $s = s(\theta, F_R)$, or only over the region with destroyed structure when $u = u(\theta, F_R)$, $s = s(\theta)$. The effects of heat liberation at the boundary of the zones are small and are not taken into account. In the present work we examine the dynamics of change of the dissipative function in the region with destroyed structure as well as in the region with intact structure. Such dependences are indispensable for the subsequent solution of the problem of heat exchange (13).

The formulated nonlinear problem (3)-(6) cannot be solved analytically. For a numerical analysis we used a discrete analog constructed by the method of control volume [7]. The results of the calculations are presented in Figs. 1-5. Figure 1 shows the time-dependent change of the tangential stress at the wall of a flat channel $\bar{\tau}_w = \tau_w/(H(\partial p/\partial z)_0) = \beta_0\tau_w/Y$. The graphs reveal the wave nature of the development of $\bar{\tau}_w$ with $\beta_0 = 0.75$, 0.6, corresponding to the "dynamic" transient regime. A larger oscillation amplitude corresponds to larger β_0. The oscillations become steady after the first period. This has to do with the considerable rate of energy dissipation in the region of flow of the destroyed structure. When $\beta_0 = 0.3$, 0.45, the stresses attain their steady level after a few oscillations. The change of stress at the channel wall is connected with the change of dimension of the near-wall zone of flow. The region with destroyed structure originates at the instant when the tangential stress at the wall becomes equal to the yield point $\bar{\tau}_w = \beta_0$. In the case of "dynamic" transient regime the region of flow increases with increasing tangential stress at the wall and decreases with its decrease. When $\bar{\tau}_w$ decreases to the level of β_0, the zone with destroyed structure vanishes. When the maximal value of the tangential stress in the subsequent period attains the yield point, then the near-wall flow develops again. When the dissipative process has reduced the amplitude of stress oscillations to values below the yield point, then a zone of destroyed structure does not originate, and the transient process comes to an end. For β_0 corresponding to the second type of transient regime the size of the zone of flow attains a steady value after the oscillations.

Figure 2a shows the dynamics of change of the flow rate of the medium $\bar{Q}$ determined in the following manner:

$$Q = 2\pi \int_{R_1}^{R_2} rv dr, \quad \bar{Q} = Q/\mu.$$

With the second type of transient process after sudden loading we find that the flow rate increases monotonically until a steady value is attained (curve 3 in Fig. 2a). At the same time near the boundaries, the medium with destroyed structure flows, and at the center the zone with intact structure moves uniformly. The abrupt drop of the pressure gradient to zero when $\bar{t}/4 = 2$ leads to the gradual disappearance of the zone of flow (Fig. 2b, c) and to purely elastic deformation of the intact structure. The values of the flow rate oscillate about zero at constant amplitude. If a "dynamic" transient regime is effected, e.g., $\beta_0 = 0.7$, 0.9 ($\delta = 1$), then the changes of flow rate are of an oscillating nature already in the first phase, viz., loading. The maximal flow rate in the first period of passage of the elastic wave exceeds the amplitude of the oscillations that is established during the subsequent periods. This is connected with the short-term effect of destruction of the structure accompanied by flow. In the case under consideration at the instant of unloading the structure is in a restored state. The drop of the pressure gradient induces a "dynamic" type of transient process as a result of which oscillations of flow rate with constant amplitude establish themselves.

An interesting feature of channels with annular profile is their modification of the "dynamic" transient regime: near the inner channel wall there are regions of flow, and next to the outer wall there are none (Fig. 2b, c). This is due to the asymmetric distribution of the tangential stresses over the cross section of the channel.

For different curvatures of a coaxially cylindrical channel we can determine from Fig. 3 those limit values of tangential stress at the wall at which destruction of the structure occurs near the inner and outer walls with the specified yield point. With large δ corresponding to a flat channel, these values coincide and are equal to $H(\partial p/\partial z)_0$.

Figure 4 presents graphs of flow rate distribution and stresses across the channel at different instants. In the case of "dynamic" transient regime (Fig. 4b, d) the region of the channel cross section can be divided into two zones according to the nature of the deformation. In the first zone, where the structure is temporarily destroyed, the wave mechanism of development of the flow rate profiles and of tangential stresses is replaced by a diffusional mechanism, and then again by a wave mechanism. In the second zone the structure is not destroyed during the entire process. Here we find only oscillations of flow rates and stresses without dissipation. During the second type of transient process we can distinguish three types of region (Fig. 4a, c). In the first region (near the channel walls) the development of the profiles proceeds by way of diffusion after the stresses have attained the yield point. In the second region, which lies at the center of the channel, the stresses and flow rates carry out attenuated oscillations about the steady values. The third region is an intermediate one, here the stresses and flow rates increase at first, like in an elastic medium, then flow begins, and the change of these variables is of a diffusional nature. Finally, when the stress level has dropped below the yield point, the flow rates and stresses after some oscillations assume equilibrium values.

Figure 5 shows the effect of the parameter ζ on the flow rate characteristics of the medium in "dynamic" regime of deformation. Smaller ζ correspond to higher intensity of the dissipative process during flow in the zone with destroyed structure. With moderate ζ ($\zeta = 5, 50$) the transient process ends toward the end of the first period because that part of the energy that is indispensable for increase of the stresses above the yield point has been dissipated. With large ζ ($\zeta = 500$), corresponding to systems with little dissipation, the transient process continues for many periods. Thus the absolute length of the transient process continues for many periods. Thus, the absolute length of the transient process with fixed time of propagation of the elastic wave T_{els} increases with increasing characteristic time of propagation of a viscous shear wave T_{vis} in a destroyed structure. It can be seen from the figure that the volume of medium passing through the channel cross section during the time of the transient process increases when ζ increases. This is due to the lower resistance when flow occurs in a region with destroyed structure since this is associated with lower intensity of energy dissipation.

Figure 6 shows the contribution of the dissipative functions, averaged over the zone of flow of the destroyed structure $\bar{\Phi}_2$ and over the region with intact structure $\bar{\Phi}_1$, to the total values of the mean dissipative function $\bar{\Phi}$. It can be seen from the graphs that with the second transient regime $\bar{\Phi}_2$ is close in value to the dissipative function $\bar{\Phi}$. Conversely, for "dynamic" transient regimes the values of the dissipative functions $\bar{\Phi}_1$ are closer to the overall dissipative function. Thus, whereas in the intact structure elastic deformations are of a purely energetic nature, with the second type of transient processes the mean stress level can always be approximately calculated over the entire width of the channel because $\bar{\Phi} \approx \bar{\Phi}_2$. In effecting the "dynamic" transient regime, the dissipative function in Eq. (13) has to be determined by the value of $\bar{\Phi}_2$. It should be emphasized that the value of $\bar{\Phi}_1$ averaged over a period of oscillations is equal to zero, and the dissipative function $\bar{\Phi}$ averaged over a period coincides with the function $\bar{\Phi}_2$ averaged over a period, therefore the problem of determining the energy equation of state of the medium is not a matter of principle.

The analysis of the transient processes of shear strain of liquid disperse systems on the basis of the elastoviscoplastic model revealed a number of peculiarities of their course. It was shown that there may be two kinds of transient regimes. The type of transient regime is determined by the parameter β_0, which depends on the yield point, the width of the channel, the jump of the pressure gradient, as well as by the curvature of the channel δ. The duration of the transient process, and also the amplitude of the oscillations of flow rates, stresses, flow-rate characteristics are correlated with the dissipation rate of mechanical energy in the zone with destroyed structure characterized by the parameter ζ. When thermal processes are modeled, it becomes necessary to take the elastic properties of the structure into account. If the balance equation of heat conduction is to be used, the nature of the reversible deformations has to be revealed. In the case of entropy and energetic elasticity the traditional balance equation of heat conduction may be used. Thus, the obtained results point up the decisive effect of structural and mechanical factors on the type of transient process and other peculiarities of the deformation of elastoviscoplastic media.

NOTATION

x, y, z, Cartesian coordinates; r, z, polar coordinates; 1(t), Heaviside's unit function; p, pressure; ρ, density; U, displacement; τ, stress in simple shear; τ_w, shear stress at the channel wall; H, width of the channel; t, time; G, elastic shear modulus; T_{els}, half-period of an elastic shear wave ($T_{els} = H\sqrt{\rho/G}$); Y, yield point; η, plastic viscosity; $\tau_{max}^{(st)}$, maximal shear stress in deformation of an elastic medium in a flat channel; $\underset{\sim}{D}$, deviator of the strain-rate tensor; $\underset{\sim}{T}$, deviator of the stress tensor; γ, shear strain; $\underset{\sim}{v}$, velocity; t_0, time of observation; $\dot{\gamma}$, shear rate; $\bar{r} = r/H$, dimensionless coordinate; $\bar{t} = t/T_{els}$, dimensionless time; ζ, β_0, β_1, dimensionless complexes; u, internal energy referred to unit mass; $\underset{\sim}{q}$, heat flux; c_p, specific heat capacity; θ, temperature; λ, thermal conductivity; A, Helmholtz' free energy; $\underset{\sim}{F}_R$, gradient of deformation relative to the reference configuration not coinciding with the configuration of the instant of observation; s, entropy; q_1, q_2, external heat fluxes on the channel walls; R_1, R_2, radii of the inner and outer cylinder, respectively, forming the coaxially cylindrical channel; $\delta = R_1/H$, parameter characterizing the curvature of the channel with constant H; h_1, h_2, width of the zone of flow at the inner and outer channel wall, respectively, $\bar{h}_1 = h_1/H$, $\bar{h}_2 = h_2/H$; μ, ν, scales of flow rate and of the dissipative function ($\mu = \pi H^2\sqrt{G/\rho}$, $\nu = \pi H G^{3/2}/\rho^{1/2}$).

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